Protokoll
Praktikum Quantenphysik

Bottle Resonator

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Contents

Evanescent Fields and Bottle Resonators

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Abstract

We research the storage of light in a Bottle Resonator, an extremely thin disconnected strand of fiber. We change the distance of a coupling fiber to the Bottle Resonator and the laser frequency and analyze the effect of distance and laser frequency on stored and transmitted power (out of the Bottle Resonator or bypassing it).

1 Introduction

1.1 Theory

1.1.1 Evanescent Fields

At the interface between two media with different optical indices, in general an incoming light wave can be reflected and/or refracted. It is possible to choose the incident angle in such a way (large angle with respect to the normal to the interface, that is a small angle with respect to the interface) that no refracted wave results from the incoming wave. However, then the electric field decays exponentially beyond the interface.

Snell's law is

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

where θ_1 and θ_2 are measured from the normal to the interface plane.

If $n_2 < n_1$, the refracted wave only exists for angles θ_1 smaller than some critical angle:

$$\sin(\theta_1) = \frac{n_2}{n_1} \sin(\theta_2)$$

$$-\arcsin\left(\frac{n_2}{n_1}\right) \leq \theta_1 \leq \arcsin\left(\frac{n_2}{n_1}\right)$$

If θ_1 is bigger than this critical angle (for $n_1 = 1.5$, $n_2 = 1$, that critical angle is $\approx 42°$), then no refracted wave into the second medium exists.

In the following, the case where $\theta_1 > 42°$ will be considered.

In particular, what happens to the part that would have been the refracted wave will be considered.

Let $E_2 = E_{02} \exp\left(i\left(\mathbf{k_2} \cdot \mathbf{r} - \omega t\right)\right)$. Split $\mathbf{k}$ (with $k_2 := |\mathbf{k_2}| = 2\pi \frac{n_2}{\lambda_0}$) into a component $k_x = k_2 \sin(\theta_2)$ parallel to the interface and a component $k_y = k_2 \cos(\theta_2) = k_2 \sqrt{1 - (\sin(\theta_2))^2} = k_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin(\theta_1)\right)^2}$ describing the propagation normal to the interface.

By precondition, $\sin(\theta_1) > \frac{n_2}{n_1}$. Therefore, the argument of the square root above is certainly negative.

$k_y = k_2 i \sqrt{\left(\frac{n_1}{n_2} \sin(\theta_1)\right)^2 - 1}$.

Let $\beta = k_2 \sqrt{\left(\frac{n_1}{n_2} \sin(\theta_1)\right)^2 - 1}$ which is real.

$$
\begin{aligned}
k_y &= i\beta \\
E_2 &= E_{02} \exp\left(i\left(k_x r_x + k_y r_y - \omega t\right)\right) \\
E_2 &= E_{02} \exp(-\beta r_y) \exp\left(i(k_x r_x - \omega t)\right)
\end{aligned}
$$

E_2 exponentially decays with distance to the interface and is called "evanescent field".

β is:

$$\beta = \frac{2\pi}{\lambda_0} \sqrt{\left(n_1 \sin(\theta_1)\right)^2 - {n_2}^2}$$

Light from one medium can be coupled into another medium if the distance between the media is small enough so that the evanescent fields overlap. This phenomenon is called "frustrated internal reflection".

1.1.2 Fabry Pérot Resonator

A Fabry Pérot Resonator consists of two mirrors, both somewhat transmissive. The mirrors are oriented so that most light is captured between them and reflected from them. The light between the two mirrors will form a (Gaussian) standing wave because of the reflection. Let l be the length of the space between the two mirrors, i be a natural number, $i > 0$, n be the refractive index of the medium, c be the speed of light. Then a standing wave is formed if:

$$l = i \frac{\lambda_0}{2\, n}$$

At the resonance frequencies ν_i there's constructive interference between the incoming light and the light between the mirrors.

$$\nu_i = i \frac{c}{2\, n\, l}$$

Let τ be the time constant of the decay of the energy between the mirrors in the absense of a driving light field, T be the reciprocal of the optical resonance frequency. Then the quality factor Q is defined as:

$$Q = 2\pi \frac{\tau}{T} = \omega_0\, \tau$$

Let N be the number of round-trips. Then the finesse $\mathcal{F}$ is defined as:

$$\mathcal{F} = 2\pi N$$

1.1.3 Bottle-Resonator Coupling

In a thin strand of optical fiber (called "Bottle Resonator") similar wave modes can also be observed. Our setup uses such an ultrathin fiber which is not connected to anything by cable. It's interesting how much energy is stored in the Bottle Resonator mode. According to [1], a system (Bottle Resonator, Light Field) is described by:

$$\frac{d}{dt}a = i\left(\omega_0 - \omega\right)a - \frac{1}{2}\left(\frac{1}{\tau_0} + \frac{1}{\tau_{fiber}}\right)a + \frac{1}{\tau_{fiber}^2}s$$

... where s is the input field amplitude and t the output field amplitude. The power transmitted through the coupling fiber is then proportional to $p^*\,p$, although the *energy* stored in the resonator is proportional to $a^*\,a$.

The transfer of optical energy between the mode of the coupling fiber and the bottle mode is described by the characteristic time constant τ_{fiber}.

The intrinsic resonator energy loss is described by τ_0.

ω_0 is the resonance frequency of the bottle mode.

In the absence of a driving light field, the equation above is solved for energy as:

$$\begin{aligned}
\frac{d}{dt}W = a^*\frac{d}{dt}a + a\frac{d}{dt}a^* &= -\left(\frac{1}{\tau_0} + \frac{1}{\tau_{fiber}}\right)W \\
\frac{d}{dt}W &= -\left(\frac{1}{\tau_0} + \frac{1}{\tau_{fiber}}\right)W \\
\frac{1}{\tau_{load}} &:= \frac{1}{\tau_0} + \frac{1}{\tau_{fiber}} \\
W = W_0 \exp &\left(-t\left(\frac{1}{\tau_{load}}\right)\right) \\
Q_{load} &:= \omega_0\,\tau_{load}
\end{aligned}$$

1.1.4 WGM in Bottle Resonator

Let there be a driving light field.

The light path of a Bottle Resonator mode (WGM) is generally circular and can have multiple rings. Q, ν_i are similar to the Fabry Pérot resonator, although l is the circumference of the fiber now. Differences are: modes with multiple rings are possible, ν_i depends on the number of rings (indirectly by i).

Let ν_0 be the optical resonance frequency of the mode, $\Delta\nu$ be the spectral linewidth. The loaded quality factor Q_{load} is [1, Section 2.1]:

$$Q_{load} = \frac{\nu_0}{\Delta\nu}$$

And the finesse is:

$$\mathcal{F} = \frac{\Delta\nu_{FSR}}{\Delta\nu}$$

1.1.5 Bottle Resonator Free Spectral Range $\Delta\nu_{FSR}$

We assume the diameter of the resonator is $60\,\overline{\text{µm}}$. Then its circumference is $l = \pi\,188.5\,\overline{\text{µm}}$.

Then $\Delta\nu_{FSR} = \frac{c}{2nl} = \frac{299792458\,\overline{\text{m/s}}}{2 \cdot n\,\pi \cdot 60\,\overline{\mu\text{m}}}$ (from the resonance condition).

Because in the Fabry-Pérot cavity, a similar equation $100\,\overline{\text{MHz}} = \frac{c}{2nL}$ holds, we can calculate $n = 1.49896$.

Next, we assume that the Fabry-Pérot Resonator and the Bottle Resonator use the same material (with the same refractive index).

Therefore, $\Delta\nu_{FSR} \approx 530.5\,\overline{\text{GHz}}$ for the Bottle Resonator.

1.1.6 Bottle Resonator Loss Mechanisms

A photon in the resonator can be lost by:

- Absorption by erbium ions (causing green fluorescence)
- Scattering on the surface of the coupling fiber (intrinsic loss)
- Scattering away at the resonator-fiber coupling point (parasitic loss)

We introduce the coupling parameter K defined by:

$$K = \frac{K_{fiber}}{K_{para} + K_0} \tag{1}$$

where $K_{fiber} = 1/\tau_{fiber}$ describes the rate of coupling between the coupling fiber mode and the resonator mode, $K_0 = 1/\tau_0$ describes the rate of coupling to intrinstic loss channels and $K_{para} = 1/\tau_{para}$ describes the rate of coupling to parasitic loss channels.

Then the probability that a photon stored in the resonator is coupled back into the fiber (rather than lost) is:

$$E_{photon} = \frac{K_{fiber}}{K_{para} + K_0 + K_{fiber}} = \frac{1}{1 + 1/K} \tag{2}$$

1.1.7 Bottle Resonator Coupling Regimes

It's common to classify different coupling regimes dependent on the ratio $\frac{\tau_0}{\tau_{fiber}}$:

- Under-coupled regime: $\tau_{fiber} > \tau_0$. For a large gap x, the transmission T_{res} is close to 1 and increases with distance. Only little power is transferred to the resonator - where it is dissipated.
- Critical coupling: $\tau_{fiber} = \tau_0$. For one particular gap the power that goes into the resonator is just as much as is lost by intrinsic losses. While the light field leaving the resonator couples back into the coupling fiber it has a phase shift of π and thus there is almost zero transmission to the coupling fiber.
- Over-coupled regime: $\tau_{fiber} < \tau_0$. For even smaller gaps x, the relative transmission T_{res} will become close to 1 again.

1.2 Experimental Setup

A monochromatic tunable laser is used as light source to feed light into both a reference Fabry Pérot Resonator and our Bottle Resonator. The bottle resonator is standalone, but is in close proximity to a coupling fiber that is transporting the light from the source. The distance between bottle resonator and coupling fiber can be varied (in one direction only) by a stepper motor (for coarse tuning) and a piezo (for fine tuning). The light is sent into the fiber in such a way that total reflection happens. Then, an evanescent wave still exists outside the (thin) coupling fiber. Because the Bottle Resonator is also very thin, the evanescent waves couple by frustrated internal reflection. (It is also possible for the wave in the Bottle Resonator to continue into the coupling fiber again, usually there will be a phase shift of π then)

We measure the power at the end of the coupling fiber using a photodiode 1. We also measure the Fabry Pérot Resonator's output power using a photodiode 2. The Fabry Pérot Resonator will be used as reference in order to be able to read frequencies off the graph.

Both results are then fed into a oscilloscope. We use a scan module to scan the frequency of the laser and trigger the oscilloscope. The horizontal axis corresponds to the laser frequency, the vertical axis corresponds to photodiode voltage. Graphs are of (1) voltage of photodiode 1, (2) voltage of photodiode 2.

Note that the goal is to judge the power stored in the Bottle Resonator, so that would be the complement of photodiode 1's signal.

Picture of the Bottle Resonator, already glowing with light from the compling fiber (which originally came from the laser):

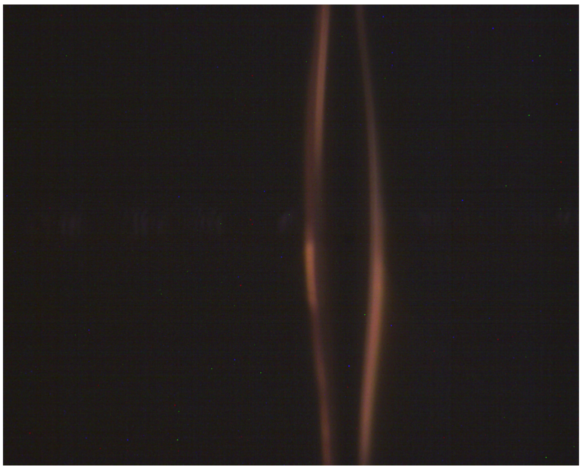


Figure 1: Bottle Resonator

2 Results

2.1 Measurements and Data Analysis

First we determined the Fabry-Pérot mode spacing. For this, for each oscillator file we did the following:

- We found the beginning first-in-time Lorenzian, t_{min}

- We found the end of the last-in-time Lorenzian, t_{max}
- We counted the number of Lorenzians (peaks), c
- We calculated the mode spacing by $n_i := (t_{max} - t_{min})/c$.

Let n be the union of all the n_i intervals. Thus the Fabry-Perot mode spacing n is (units will cancel through the division g/n but they are formally seconds):

$$0.001067 \leq n \leq 0.001127$$

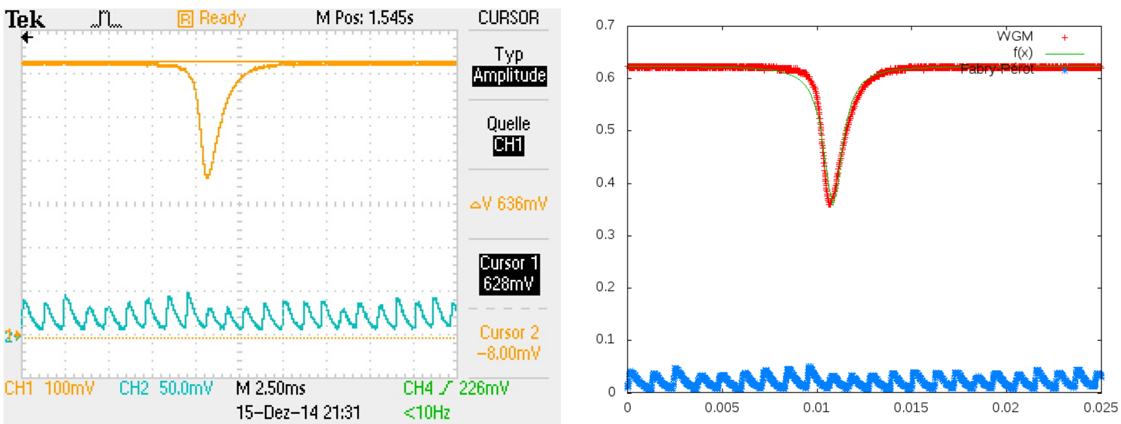
The value n corresponds to the known Fabry-Perot mode spacing $100\,\overline{\text{MHz}}$ and we later use it to convert the g linewidth of the bottle resonator to $\overline{\text{MHz}}$.

2.1.1 Critical Coupling

For Critical Coupling, the transmission comes closest to zero because all the power is dissipated in the resonator. The light leaving the resonator couples back into the coupling fiber but is phase-shifted by π.

The corresponding quality factor is:

$$Q_{crit} = \frac{Q_0}{2}$$



$$g = 118.403 \cdot 10^{-5} \pm 8.07 \cdot 10^{-6},\ \frac{\Delta g}{g} = (0.6816\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\,g/n = (108 \pm 3.7)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 3.26 \cdot 10^6 \pm 1.1 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 4.92 \cdot 10^3 \pm 1.7 \cdot 10^2$$

Figure 2: File 58

For critical coupling, we determined the linewidth $\Delta\nu$, loaded quality factor $Q_{load} = \frac{\nu_0}{\Delta\nu}$ (where ν_0 is the mode resonance frequency which we approximated using $c = \nu_0\lambda_0$ with $\lambda_0 = 852\,\overline{\text{nm}}$, one of the used wavelengths of our laser. We assumed that the extra error by not using a more variable wavelength is negilible).

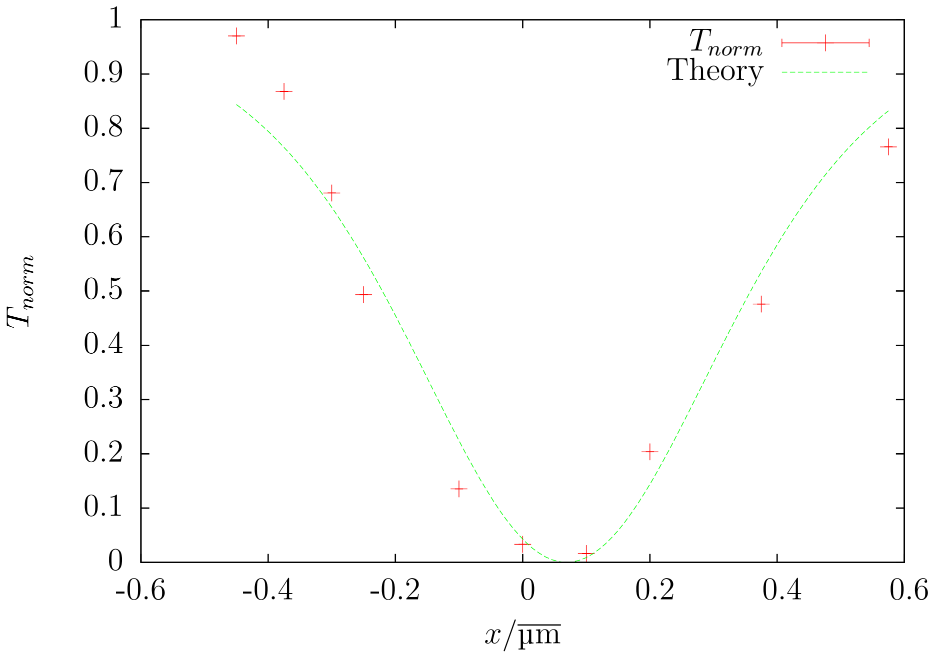
We determine the intrinsic quality factor Q_0 by $Q_0 = 2\,Q_{load}$:

$$Q_0 = 6520000 \pm 110000$$

2.1.2 Resonant Transmission vs Coupling Gap

Next we studied how transmission changes with gap, starting at a distance of $1.3\,\overline{\mu\text{m}}$, moving to less distance. The x coordinates in the following plots are relative to $13.85\,\overline{\mu\text{m}}$ and using only Measurement 2 (we did not plot Measurement 1 because it is extremely similar and the x coordinate ranges are overlapping). The conversion from piezo voltages to distances used was: $\Delta x = \Delta U_{piezo}\,0.25\,\overline{\mu\text{m}}/\overline{\text{V}}$.

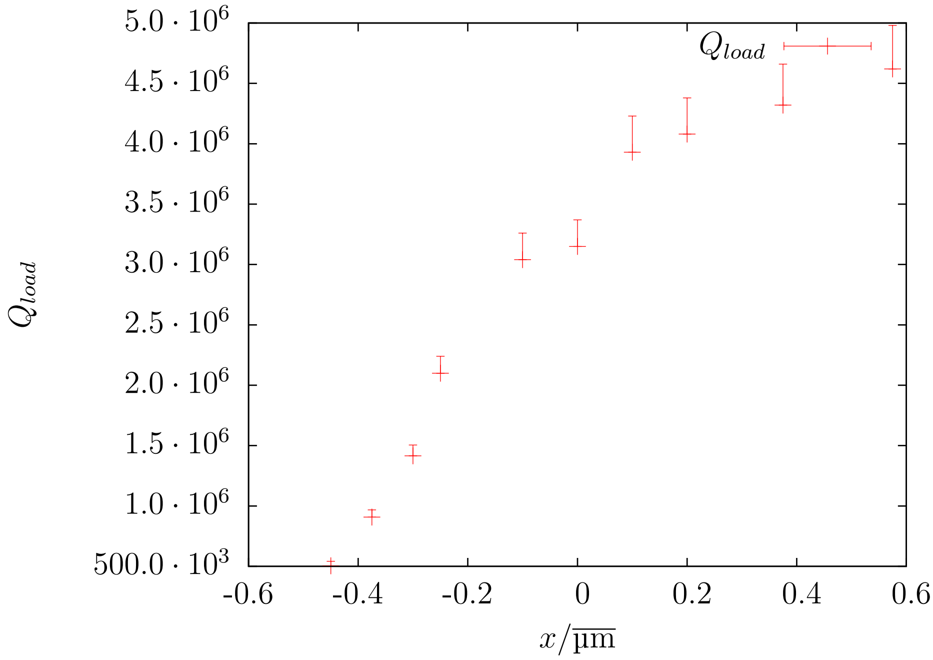
Our dips were quite small (dip height 40% of the maximum signal). Therefore, we renormalized the data for T_{res} to the interval 0..1. We call the normalized data T_{norm}.



Fitting this to the theroetically expected formula gave us large uncertainities:

$$a := 1.586, a_{min} := -8.065 \cdot 10^6, a_{max} := 8.065 \cdot 10^6$$
$$b := -0.006795, b_{min} := -8.349 \cdot 10^5, a_{max} := 8.349 \cdot 10^5$$
$$c := 0.1642, c_{min} := 0.1403, c_{max} := 0.188$$
$$T_{norm}(x) = \left(\frac{1 - a\,\exp(-(x-b)/c)}{1 + a\,\exp(-(x-b)/c)}\right)^2$$

2.1.3 Quality factor vs. Coupling Gap



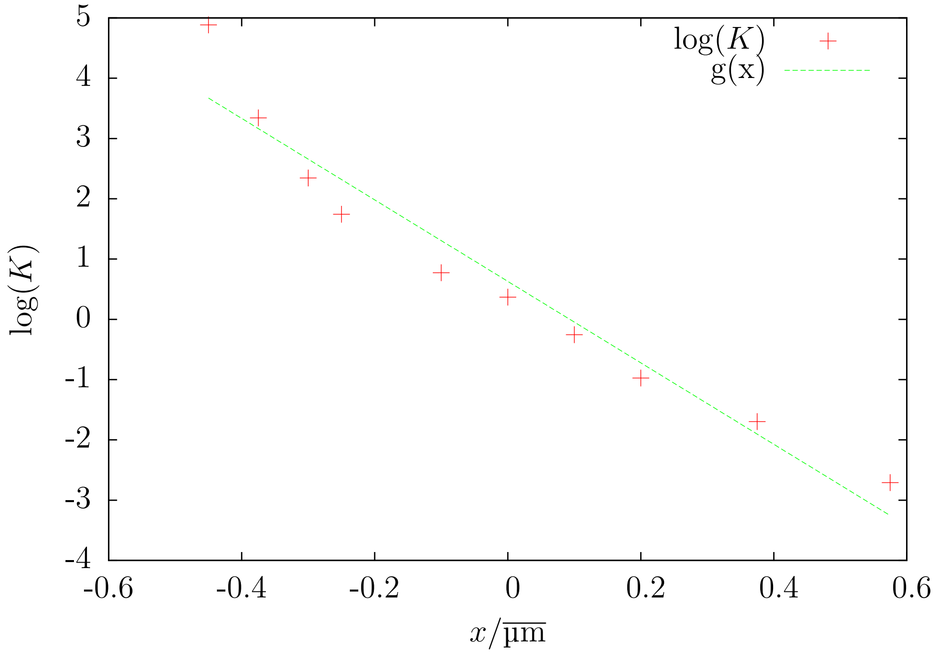
2.1.4 Photon Coupling Efficiency

For the overcoupled regime:

$$K = \frac{1 + \sqrt{T_{norm}}}{1 - \sqrt{T_{norm}}}$$

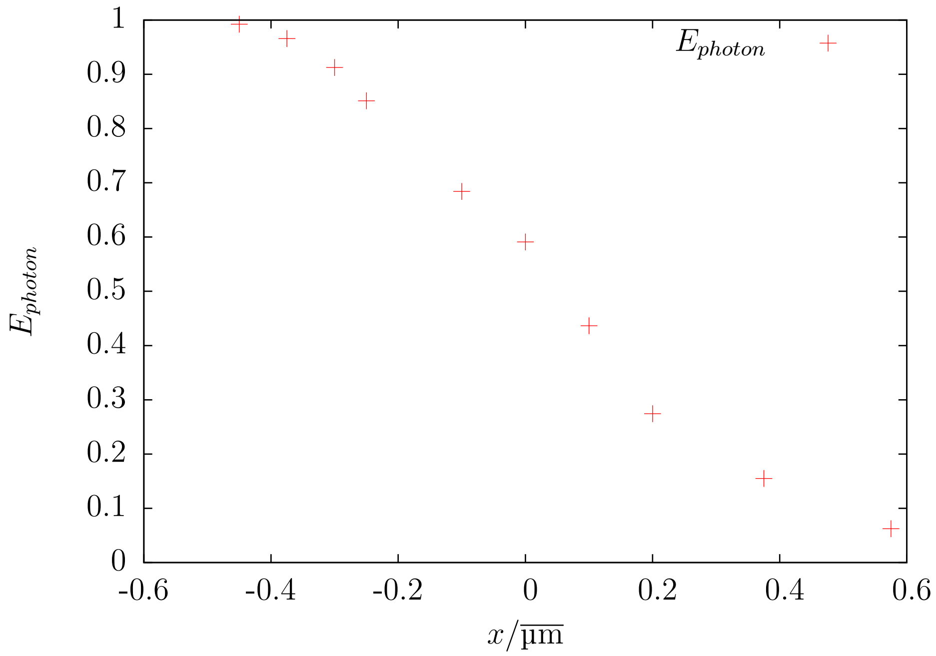
For the undercoupled regime:

$$K = \frac{1 - \sqrt{T_{norm}}}{1 + \sqrt{T_{norm}}}$$



$log(K)$ was fit with $g(x) := a\,x + b\,\overline{\mu m}$, with $a = -6.75751 \pm 0.5784$, $b = 0.630094 \pm 0.1854$.

$E_{photon} = \frac{1}{1+1/K}$.



2.2 Conclusion and Summary

We reproduced the expected linear dependency of $\log(K)$ (with K being the photon coupling efficiency) vs x.

The mode we used did not couple that well, our dips were just 40% of the maximum, therefore our analysis has some extra error.

More sources of error:

- absorption
- scattering at the beam splitter
- scattering at surface blemishes, dust

3 Appendix

Laser was set up with:

Name	Value
Temperature	$41.6\,\overline{°C}$
Wavelength	$852\,\overline{\text{nm}}$
Current	$50.7\,\overline{\text{mA}}$
Scanning Amplitude	5.1

Fabry-Pérot-Frequency $= 100\,\overline{\text{MHz}}$.
Fabry-Pérot-Cavity-Length $L := 1\,\overline{\text{m}}$

3.1 Fit Function

The function used for fitting the Lorentzian in the mode spectra (see appendix 3.2, appendix 3.3) was proportional to (1 − Lorentzian):

$$f(x) := \begin{cases} 0, g < 0 \\ a + b\left(1 - \frac{1}{\pi}\frac{g/2}{(x-x_0)^2+(g/2)^2}\right), g \geq 0 \end{cases}$$

$$f(x_0) = \begin{cases} 0, g < 0 \\ a + b\left(1 - \frac{1}{\pi}\frac{1}{g/2}\right), g \geq 0 \end{cases}$$

The center height h of the transmission dip is thus approximately:

$$h := f(\infty) - f(x_0)$$

$$h \approx \frac{b}{\pi\, g/2}$$

3.2 Measurement 1

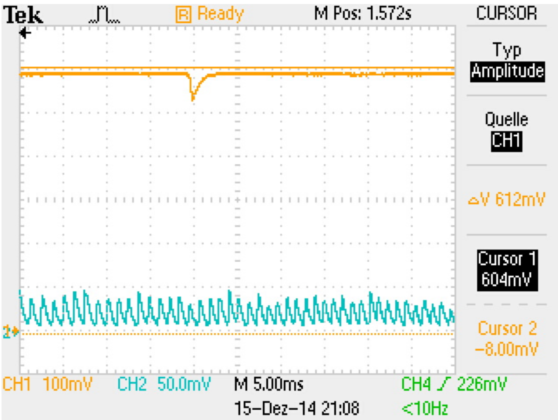
Stepper position $= 44.0002\,\overline{\mu m}$
With polarization 0, 0.

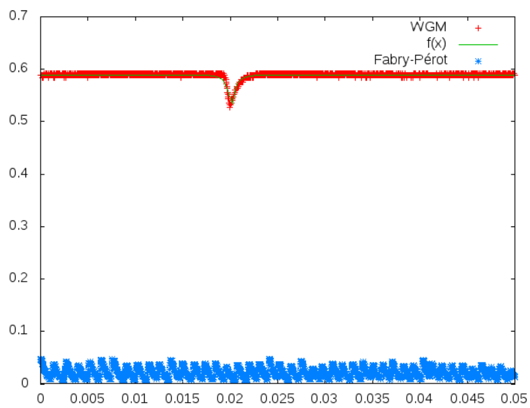
$$60\,\overline{V} = 15\,\overline{\mu m}$$

$$\overline{V} = 0.25\,\overline{\mu m}$$

Time	Piezopos/$\overline{V}$	Piezopos/$\overline{\mu m}$ (calc.)	File
21:08	48.0	12.0	44
21:10	47.4	11.85	45
21:11	46.8	11.7	46
21:11	46.4	11.6	47
21:13	46.0	11.5	48
21:14	45.6	11.4	49
21:15	45.2	11.3	50
21:17	45.0	11.25	51
21:18	44.6	11.15	52
21:19	44.3	11.075	53

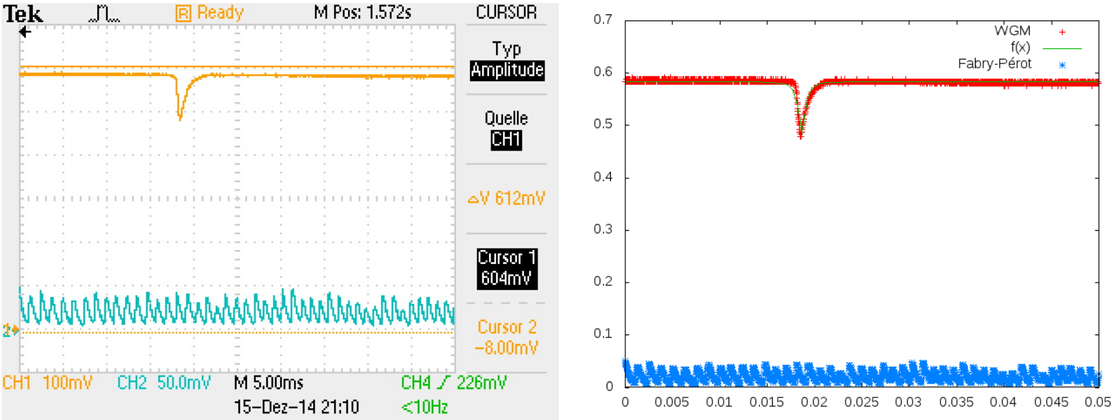
The FWHM ($\Delta\nu$) of the WGM mode (CH1) is as follows:





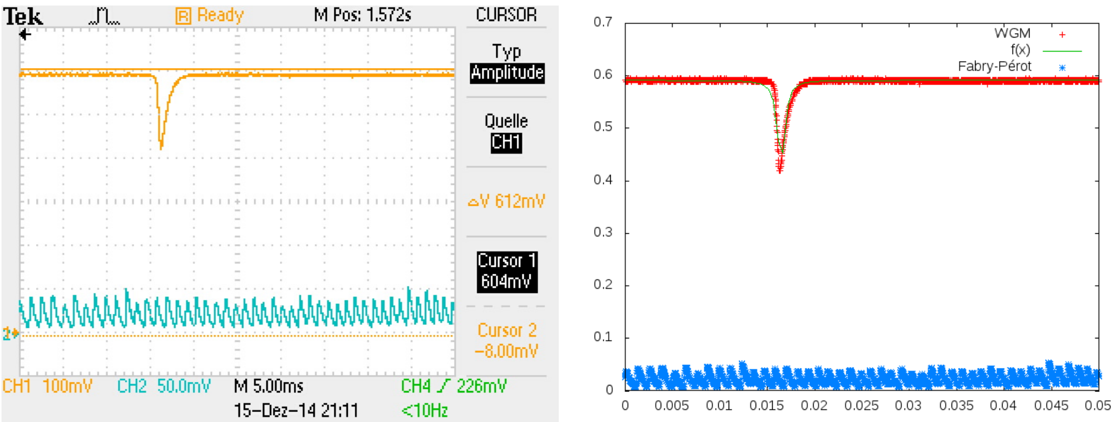
$$g = 79.2308 \cdot 10^{-5} \pm 1.31 \cdot 10^{-5}, \frac{\Delta g}{g} = (1.654\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\, g/n = (72.3 \pm 3.2)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 4.88 \cdot 10^6 \pm 2.1 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 7.35 \cdot 10^3 \pm 3.2 \cdot 10^2$$

Figure 3: File 44



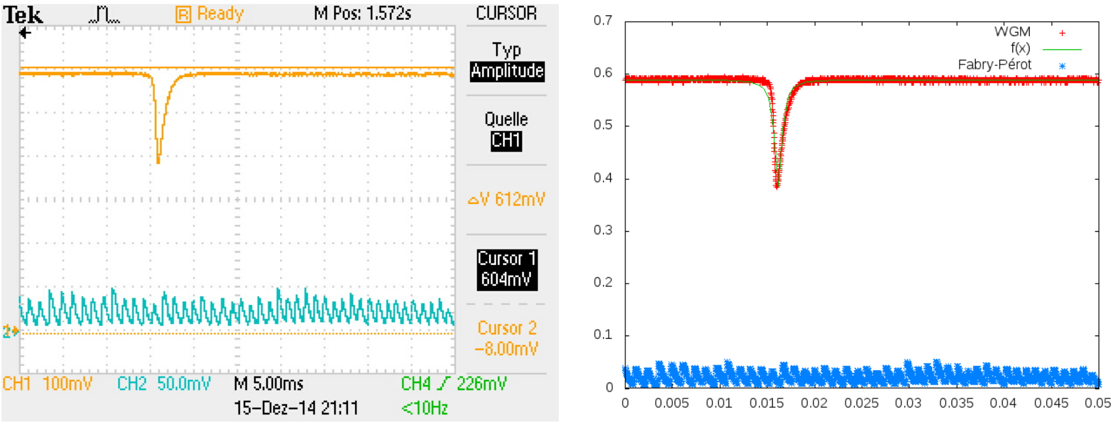
$$g = 80.618 \cdot 10^{-5} \pm 1.105 \cdot 10^{-5}, \frac{\Delta g}{g} = (1.371\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\, g/n = (73.6 \pm 3)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 4.79 \cdot 10^6 \pm 2 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 7.22 \cdot 10^3 \pm 3 \cdot 10^2$$

Figure 4: File 45



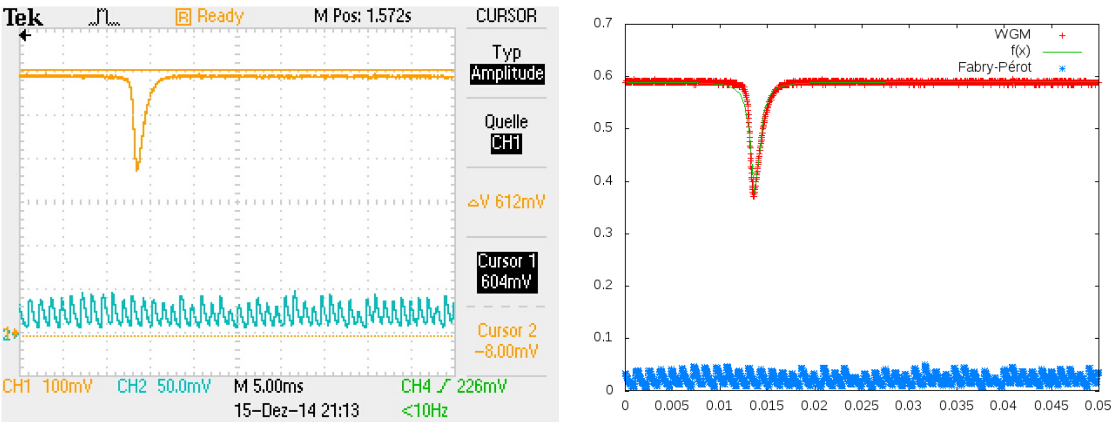
$$g = 88.6968 \cdot 10^{-5} \pm 9.348 \cdot 10^{-6}, \frac{\Delta g}{g} = (1.054\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\, g/n = (80.9 \pm 3.1)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 4.35 \cdot 10^6 \pm 1.6 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 6.56 \cdot 10^3 \pm 2.5 \cdot 10^2$$

Figure 5: File 46



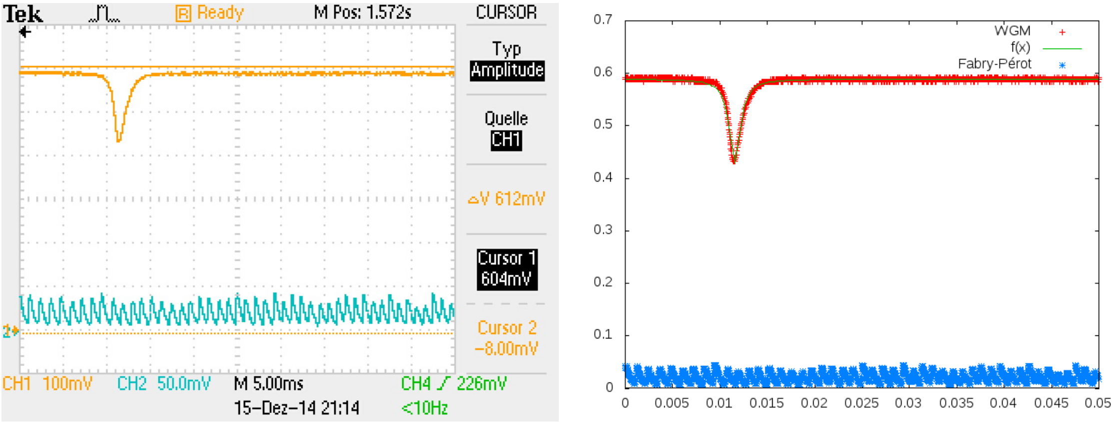
$$g = 93.3021 \cdot 10^{-5} \pm 8.961 \cdot 10^{-6}, \frac{\Delta g}{g} = (0.9605\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\,g/n = (85.1 \pm 3.1)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 4.14 \cdot 10^6 \pm 1.5 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 6.24 \cdot 10^3 \pm 2.3 \cdot 10^2$$

Figure 6: File 47



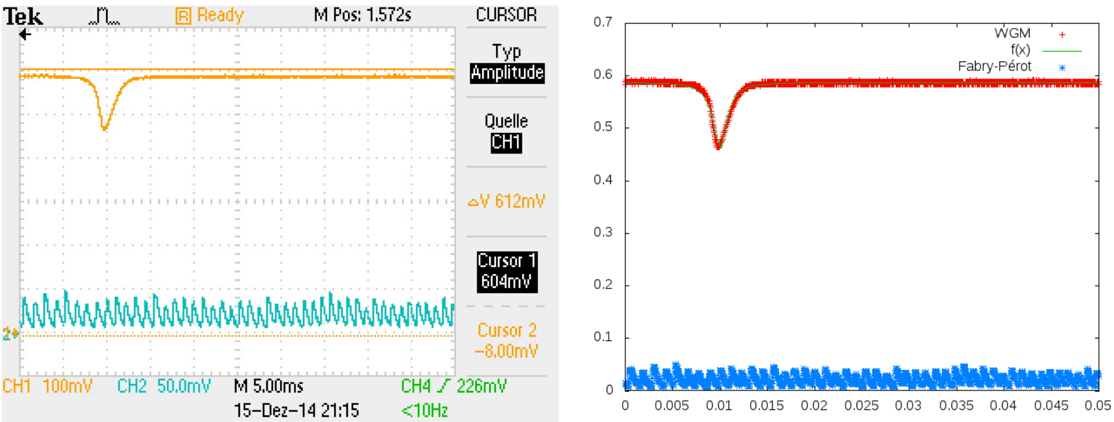
$$g = 111.514 \cdot 10^{-5} \pm 9.395 \cdot 10^{-6}, \frac{\Delta g}{g} = (0.8425\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\,g/n = (102 \pm 3.6)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 3.46 \cdot 10^6 \pm 1.2 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 5.22 \cdot 10^3 \pm 1.9 \cdot 10^2$$

Figure 7: File 48



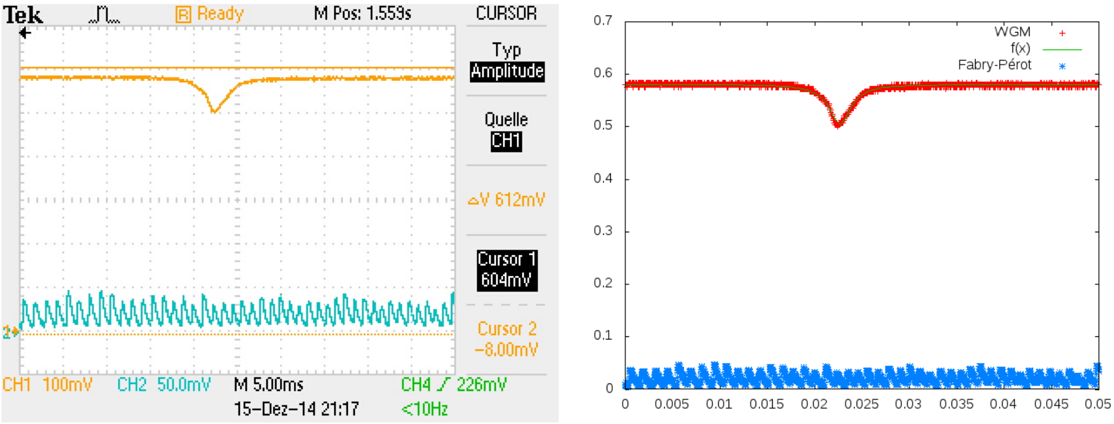
$$g = 139.057 \cdot 10^{-5} \pm 7.996 \cdot 10^{-6}, \; \frac{\Delta g}{g} = (0.575\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\, g/n = (127 \pm 4.2)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 2.78 \cdot 10^6 \pm 9.2 \cdot 10^4$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 4.19 \cdot 10^3 \pm 1.4 \cdot 10^2$$

Figure 8: File 49



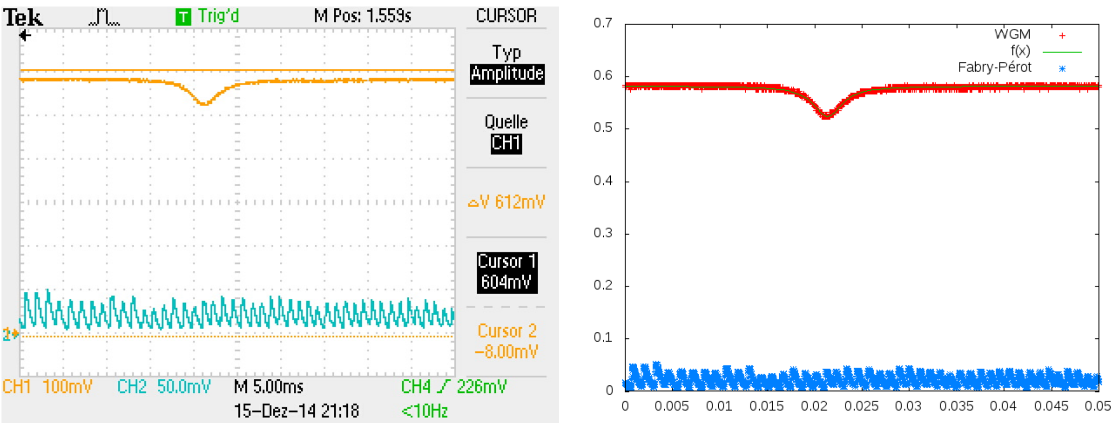
$$g = 175.918 \cdot 10^{-5} \pm 1.135 \cdot 10^{-5}, \; \frac{\Delta g}{g} = (0.645\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\, g/n = (161 \pm 5.4)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 2.19 \cdot 10^6 \pm 7.4 \cdot 10^4$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 3.31 \cdot 10^3 \pm 1.1 \cdot 10^2$$

Figure 9: File 50



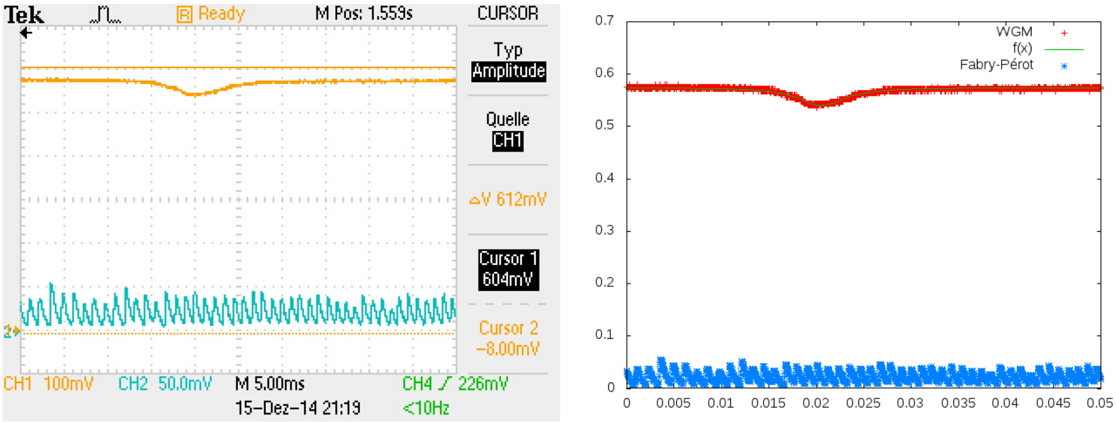
$$g = 272.79 \cdot 10^{-5} \pm 1.668 \cdot 10^{-5},\ \frac{\Delta g}{g} = (0.6115\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\, g/n = (249 \pm 8.3)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 1.42 \cdot 10^6 \pm 4.7 \cdot 10^4$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 2.13 \cdot 10^3 \pm 71$$

Figure 10: File 51



$$g = 363.512 \cdot 10^{-5} \pm 2.604 \cdot 10^{-5},\ \frac{\Delta g}{g} = (0.7165\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\, g/n = (332 \pm 11)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 1.06 \cdot 10^6 \pm 3.7 \cdot 10^4$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 1.6 \cdot 10^3 \pm 55$$

Figure 11: File 52



$$g = 600.628 \cdot 10^{-5} \pm 6.532 \cdot 10^{-5}, \frac{\Delta g}{g} = (1.087\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\, g/n = (548 \pm 21)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 6.43 \cdot 10^5 \pm 2.5 \cdot 10^4$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 969 \pm 37$$

Figure 12: File 53

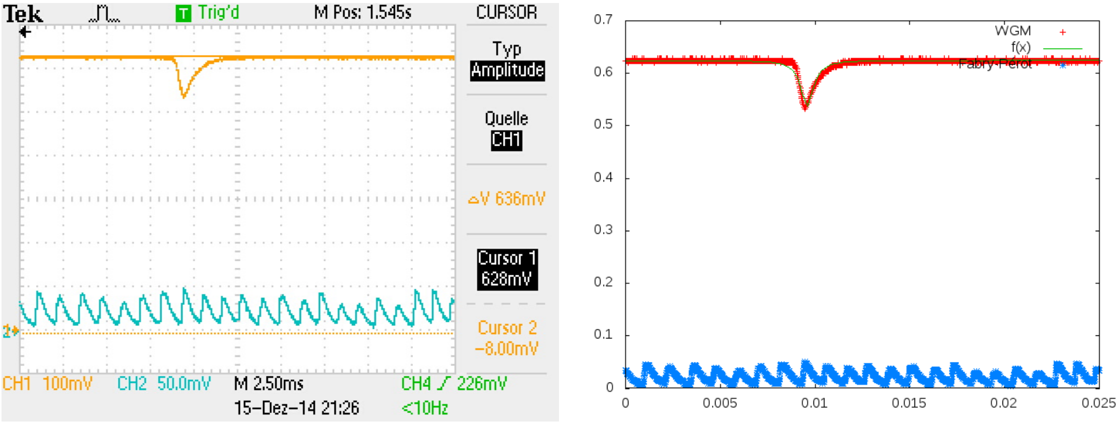
3.3 Measurement 2

Stepper position $= 42.2699\,\overline{\mu m}$

With polarization 210 for the $\lambda/2$ polarisator and 140 for the $\lambda/4$ polarisator in order to maximize dip height:

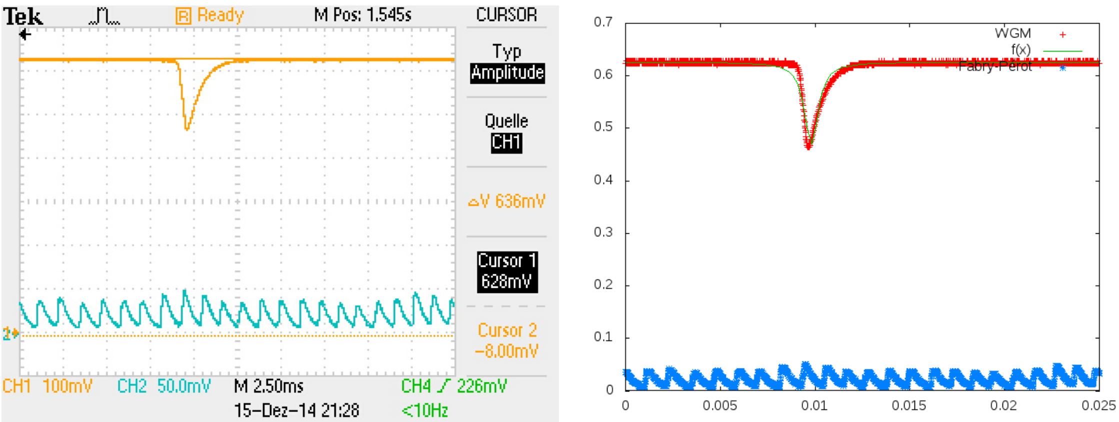
Time	Piezopos/$\overline{V}$	Piezopos/$\overline{\mu m}$ (calc.)	File
21:26	57.7	14.425	54
21:28	56.9	14.225	55
21:29	56.2	14.05	56
21:30	55.8	13.95	57
21:31	55.4	13.85	58 (crit.)
21:32	55.0	13.75	59
21:33	54.4	13.6	60
21:34	54.2	13.55	61
21:35	53.9	13.475	62
21:36	53.6	13.4	63
21:38	53.3	13.325	- (touches)

The FWHM ($\Delta\nu$) of the WGM mode (CH1) is as follows:



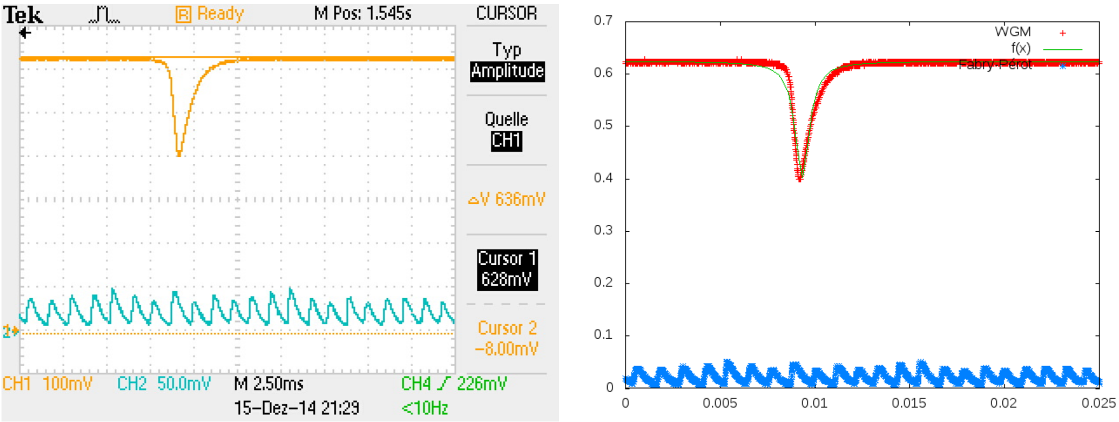
$$g = 80.3815 \cdot 10^{-5} \pm 8.37 \cdot 10^{-6}, \frac{\Delta g}{g} = (1.041\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\, g/n = (73.3 \pm 2.8)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 4.8 \cdot 10^6 \pm 1.8 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 7.24 \cdot 10^3 \pm 2.7 \cdot 10^2$$

Figure 13: File 54



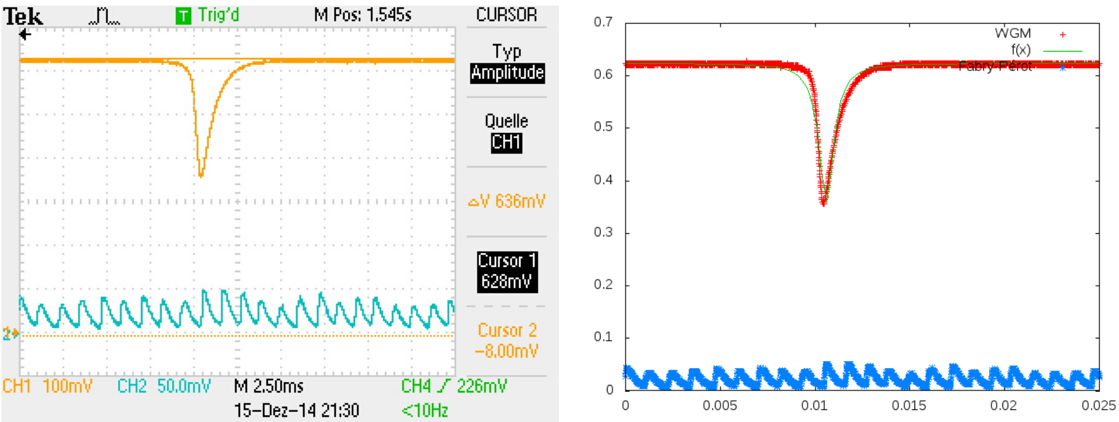
$$g = 86.0937 \cdot 10^{-5} \pm 9.09 \cdot 10^{-6}, \frac{\Delta g}{g} = (1.056\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\, g/n = (78.6 \pm 3)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 4.49 \cdot 10^6 \pm 1.7 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 6.76 \cdot 10^3 \pm 2.6 \cdot 10^2$$

Figure 14: File 55



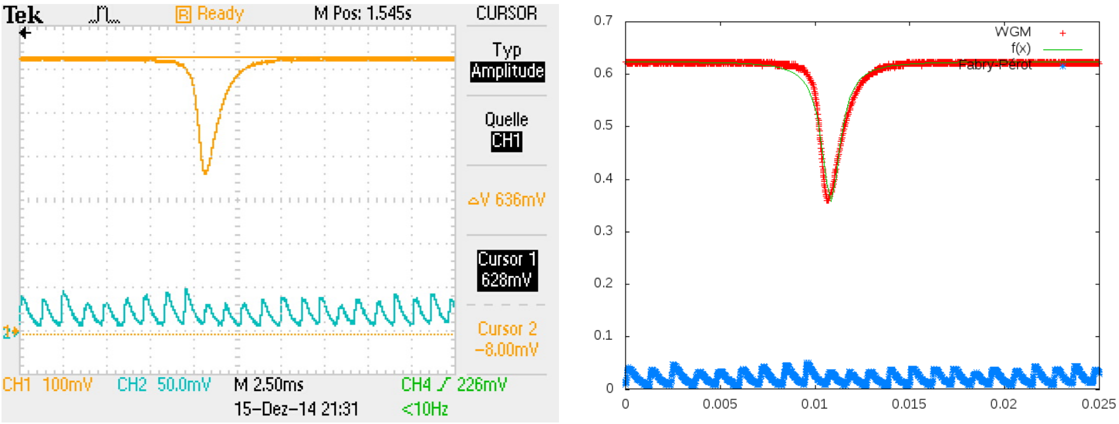
$$g = 91.3406 \cdot 10^{-5} \pm 8.472 \cdot 10^{-6}, \frac{\Delta g}{g} = (0.9275\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\, g/n = (83.3 \pm 3.1)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 4.23 \cdot 10^6 \pm 1.5 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 6.37 \cdot 10^3 \pm 2.3 \cdot 10^2$$

Figure 15: File 56



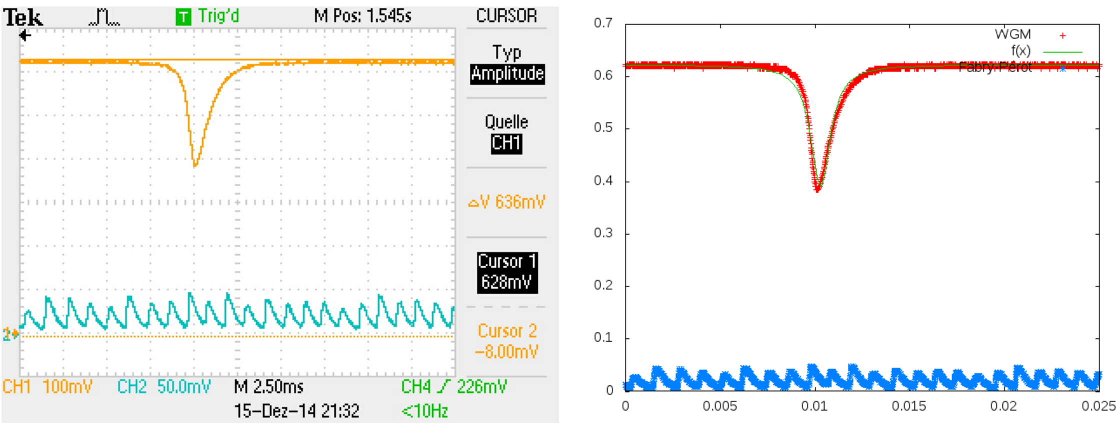
$$g = 94.6162 \cdot 10^{-5} \pm 8.288 \cdot 10^{-6}, \frac{\Delta g}{g} = (0.8759\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\, g/n = (86.3 \pm 3.1)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 4.08 \cdot 10^6 \pm 1.5 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 6.15 \cdot 10^3 \pm 2.2 \cdot 10^2$$

Figure 16: File 57



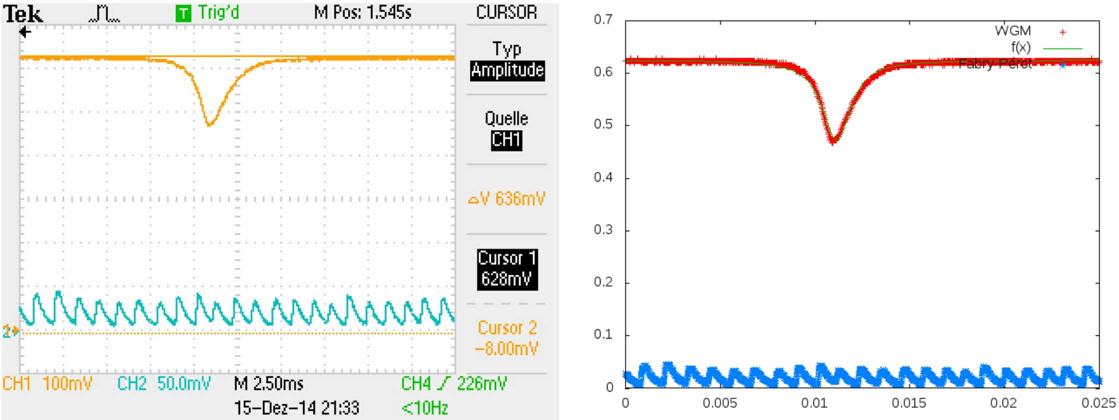
$$g = 118.403 \cdot 10^{-5} \pm 8.07 \cdot 10^{-6},\ \frac{\Delta g}{g} = (0.6816\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\,g/n = (108 \pm 3.7)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 3.26 \cdot 10^6 \pm 1.1 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 4.92 \cdot 10^3 \pm 1.7 \cdot 10^2$$

Figure 17: File 58



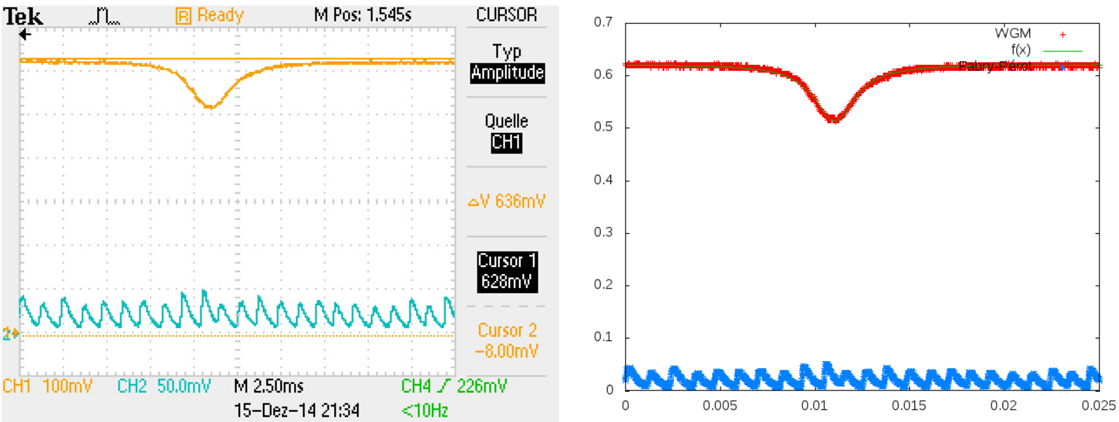
$$g = 122.676 \cdot 10^{-5} \pm 8.022 \cdot 10^{-6},\ \frac{\Delta g}{g} = (0.6539\%)$$
$$\Delta\nu := 100\,\overline{\mathrm{MHz}}\,g/n = (112 \pm 3.8)\,\overline{\mathrm{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 3.15 \cdot 10^6 \pm 1.1 \cdot 10^5$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 4.75 \cdot 10^3 \pm 1.6 \cdot 10^2$$

Figure 18: File 59



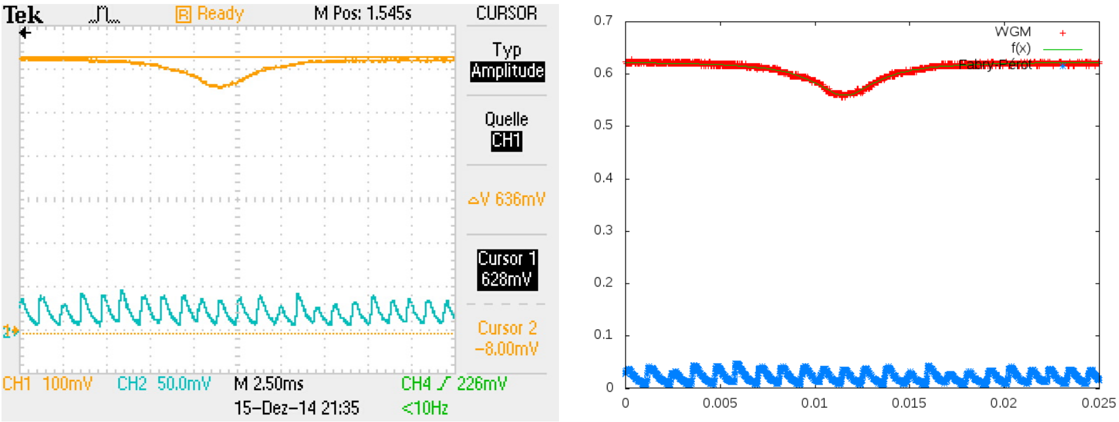
$$g = 177.882 \cdot 10^{-5} \pm 8.509 \cdot 10^{-6},\ \frac{\Delta g}{g} = (0.4784\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\,g/n = (162 \pm 5.2)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 2.17 \cdot 10^6 \pm 7 \cdot 10^4$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 3.27 \cdot 10^3 \pm 1.1 \cdot 10^2$$

Figure 19: File 60



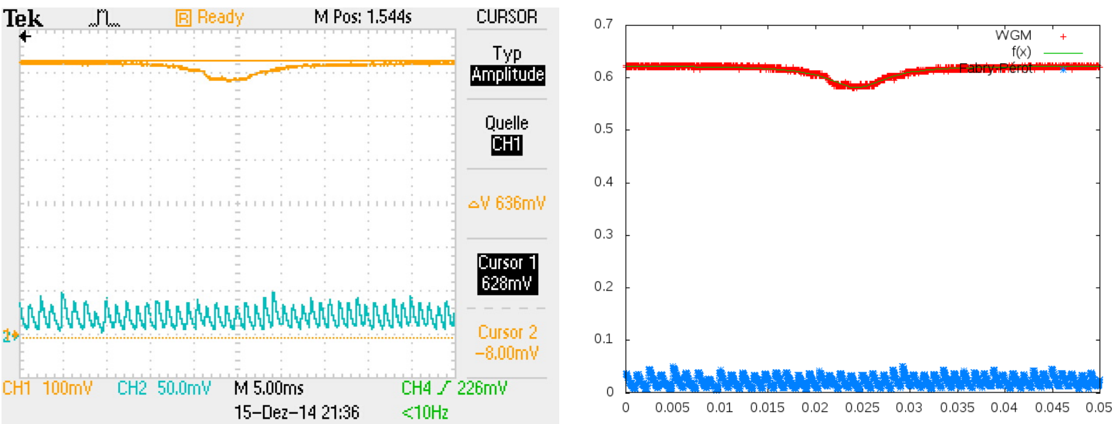
$$g = 264.126 \cdot 10^{-5} \pm 9.639 \cdot 10^{-6},\ \frac{\Delta g}{g} = (0.3649\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\,g/n = (241 \pm 7.5)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 1.46 \cdot 10^6 \pm 4.5 \cdot 10^4$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 2.2 \cdot 10^3 \pm 68$$

Figure 20: File 61



$$g = 411.414 \cdot 10^{-5} \pm 1.996 \cdot 10^{-5},\ \frac{\Delta g}{g} = (0.485\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\,g/n = (375 \pm 12)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 9.38 \cdot 10^5 \pm 3 \cdot 10^4$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 1.41 \cdot 10^3 \pm 46$$

Figure 21: File 62



$$g = 738.194 \cdot 10^{-5} \pm 6.394 \cdot 10^{-5},\ \frac{\Delta g}{g} = (0.8662\%)$$
$$\Delta\nu := 100\,\overline{\text{MHz}}\,g/n = (674 \pm 24)\,\overline{\text{MHz}}$$
$$Q_{load} := \frac{\nu_0}{\Delta\nu} = 5.23 \cdot 10^5 \pm 1.9 \cdot 10^4$$
$$\mathcal{F} := \frac{\Delta\nu_{FSR}}{\Delta\nu} = 789 \pm 28$$

Figure 22: File 63

References

[1] Prof. Dr. Arno Rauschenbeutel: Evanescent fields & Bottle resonators