Protokoll
Praktikum Quantenphysik

Noise Fundamentals

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Contents

1 Introduction

We determine the Boltzmann constant by examining Johnson noise.

1.1 Theory

1.1.1 Johnson Noise

All circuits exhibit noise. Different sources of noise exist:

- Interference: Presence of an (unwanted) signal which is added to the desired signal.
- Technical Noise: Noise that exists only because of faults in our measurement apparatus.
- Fundamental Noise: Noise that is intrinsic to our measurement apparatus and cannot be removed.

We are interested in measuring fundamental noise, more specifically in Johnson Noise. Johnson Noise was first theoretically described by Nyquist. Johnson Noise is white noise similar to blackbody radiation, just limited to one spatial dimension, along a transmission line.

The available spectral power density of Johnson Noise between two resistors separated by a transmission line to a first approximation is P_f as follows (similar to Rayleigh-Jeans - modes' boundary conditions force that an integer multiple of half the mode's wavelength fits on the transmission line)[2]:

$$P_f = k_B T$$
$$P_J := \int df k_B T = k_B T(f_2 - f_1) = k_B T \Delta f$$

A better analysis takes into account that an ultraviolet catastrophe does not happen (but would have to in this description). Similar to Planck one gets:

$$\beta := \frac{1}{k_B T}$$
$$P_f = k_B T \frac{\beta h f}{\exp(\beta h f) - 1}$$
$$P_f = h f \frac{1}{\exp(\beta h f) - 1}$$

The limiting case is contained if $\beta h f << 1$ (Taylor series of the exponential):

$$P_f \approx h f \frac{1}{1 + \beta h f - 1}$$
$$P_f \approx h f \frac{1}{\beta h f}$$
$$P_f \approx h f \frac{k_B T}{h f}$$
$$P_f \approx k_B T$$
$$P_J \approx k_B T \Delta f$$

This power is available (like a voltage source in series) to the noisy resistor and the measurement apparatus.

$$P_J = {U_0}^2/R = {U_0}^2/R$$

Let U_J be the voltage across our noisy resistor, R be the resistance ($\pm 0.1\% R$) of it, k_B be the Boltzmann constant, T be the absolute temperature ($\pm 5K$).

For an open load on the transmission line, the available voltage is twice U_0:

$$U_J = 2U_0$$
$${U_J}^2 = 4{U_0}^2$$
$$\langle {U_J}^2/R \rangle = 4P_J = 4k_B T \Delta f$$
$$\langle {U_J}^2 \rangle = 4k_B R T \Delta f$$

We thus arrived at Johnson Noise:

$\langle {U_J}^2 \rangle = 4k_B R T \Delta f$ (Nyquist's formula)[1]

$\langle U_J \rangle = 0$

1.1.2 Amplifier Noise

Unfortunately, our measurement also contains amplifier noise which we eventually have to subtract.

Amplifier noise:

Assumed to be independent of R.

Assume: $\langle U_N \rangle = 0$

Total noise (by assumption):

$\langle U^2 \rangle := \langle {U_J}^2 \rangle + \langle {U_N}^2 \rangle$

$\langle U \rangle := 0$

1.1.3 Effective Bandwidth

The filters used do not cut off at exactly the corner frequencies but rather weaken the signal steadily starting from there going outside. In order to be able to use the Nyquist formula one has to first find an equivalent "sharp" filter which would pass the same noise power as the real filter and use its corner frequencies to calculate Δf.

f_1/Hz	$f_2 = 0.33kHz$ $G_2 = 10000$ $\Delta f/Hz$	$f_2 = 1kHz$ $G_2 = 10000$ $\Delta f/Hz$	$f_2 = 3.3kHz$ $G_2 = 5000$ $\Delta f/Hz$	$f_2 = 10kHz$ $G_2 = 3000$ $\Delta f/Hz$	$f_2 = 33kHz$ $G_2 = 1500$ $\Delta f/Hz$	$f_2 = 100kHz$ $G_2 = 1000$ $\Delta f/Hz$
30	333 ± 14	$(1.08\pm 0.043)10^3$	$(3.63\pm 0.146)10^3$	$(1.11\pm 0.0443)10^4$	$(3.66\pm 0.22)10^4$	$(1.11\pm 0.0888)10^5$
100	258 ± 11	$(1\pm 0.04)10^3$	$(3.55\pm 0.143)10^3$	$(1.1\pm 0.044)10^4$	$(3.65\pm 0.219)10^4$	$(1.11\pm 0.0888)10^5$
300	105 ± 5	784 ± 32	$(3.33\pm 0.134)10^3$	$(1.08\pm 0.0431)10^4$	$(3.63\pm 0.218)10^4$	$(1.11\pm 0.0886)10^5$
1000	9 ± 1	278 ± 12	$(2.58\pm 0.104)10^3$	$(1\pm 0.04)10^4$	$(3.55\pm 0.213)10^4$	$(1.1\pm 0.088)10^5$
3000	0.4 ± 0.02	28 ± 2	$(1.05\pm 0.043)10^3$	$(7.84\pm 0.314)10^3$	$(3.33\pm 0.2)10^4$	$(1.08\pm 0.0862)10^5$

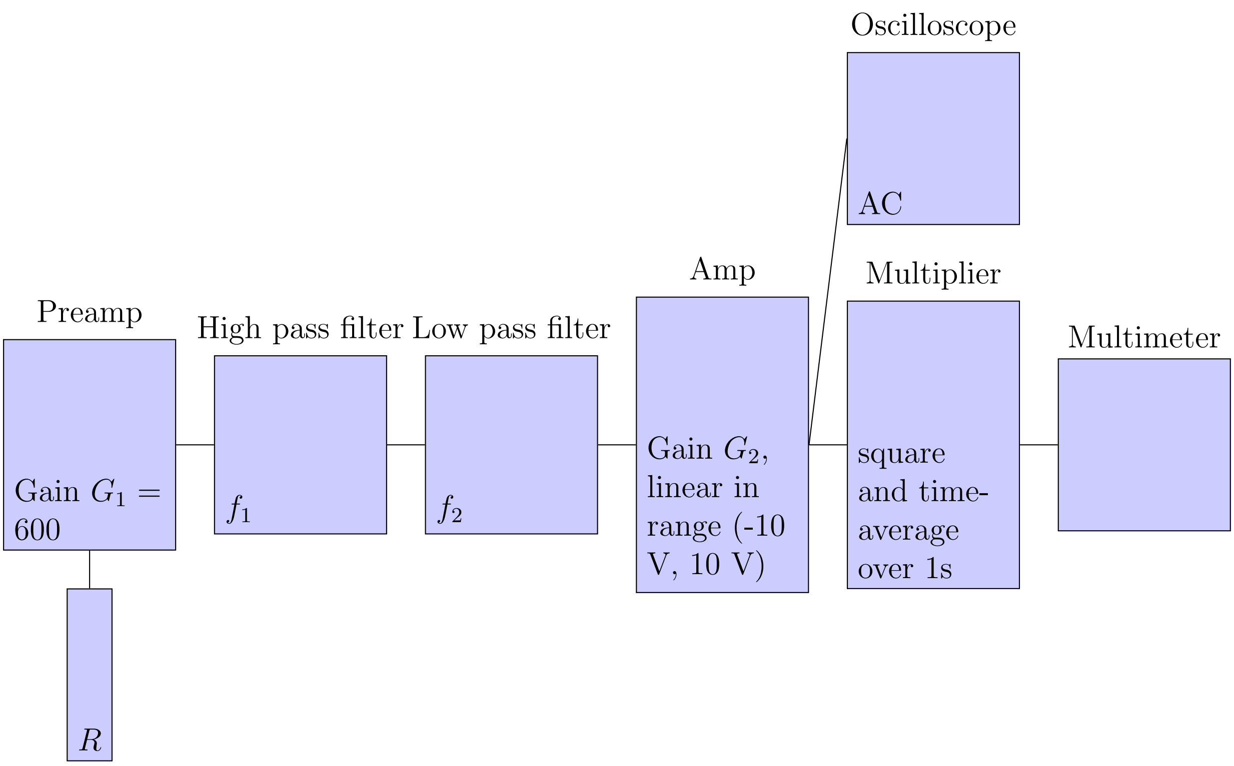
Table 1: Effective Noise Bandwidth

Uncertainities in Δf are ($\pm 4\%$), except for the rightmost column where they are ($\pm 8\%$) and for the next-to-rightmost column where they are ($\pm 6\%$).

Source: [1, Section 1.5].

1.2 Experimental Setup

1.2.1 Connections



A (changeable) resistor R (with tolerance ($\pm 1\% R$)) is connected to a preamplifier (with gain $G_1 = 600$) which is connected to filters which is connected to the main amplifier (gain G_2) which is connected to a multiplier (which is used for squaring and time-averaging the result over 1s) which is then connected to a digital multimeter. After the resistor is set up, the noise in it is then measured using the digital multimeter. We use AC coupling in order to minimize error (influence of DC offset).

1.2.2 Influence of Gain on Measured Values

Let G_1 be the preamplifier gain, G_2 be the amplifier gain (see table below), U_{met} be the measured (amplified) noise in:

$$\begin{aligned}
G_1 &:= 600 \\
G &:= \frac{G_1 G_2}{\sqrt{10V}} \\
U_{met} &:= G^2 \langle U^2 \rangle \\
U_{met0} &:= G^2 \langle {U_N}^2 \rangle \\
\Delta U_{met} &:= U_{met} - U_{met0}
\end{aligned}$$

1.2.3 Determination of Boltzmann's Constant

We are interested in k_B, so first rearrange to get access to $\langle U^2 \rangle$, then substitute in order to get $\langle {U_J}^2 \rangle$, then substitute in order to get k_B.

$$\frac{U_{met}}{G^2} = \langle U^2 \rangle$$
$$\frac{U_{met}}{G^2} = \langle {U_J}^2 \rangle + \langle {U_N}^2 \rangle$$
$$\frac{U_{met}}{G^2} - \langle {U_N}^2 \rangle = \langle {U_J}^2 \rangle$$
$$\frac{1}{G^2} \left(U_{met} - U_{met0} \right) = \langle {U_J}^2 \rangle$$
$$\frac{1}{4RTG^2 \Delta f} \left(U_{met} - U_{met0} \right) = k_B$$

2 Results

2.1 Measurements and Analysis

2.1.1 Correcting for Amp Noise

First we measured just the amp noise (equal in first approximation to $U_{met} = U_{met0}$ measured on a resistor with $R = 1\Omega$).

f_1/Hz	$f_2 = 0.33kHz$ $G_2 = 10000$ U_{met0}/V	$f_2 = 1kHz$ $G_2 = 10000$ U_{met0}/V	$f_2 = 3.3kHz$ $G_2 = 5000$ U_{met0}/V	$f_2 = 10kHz$ $G_2 = 3000$ U_{met0}/V	$f_2 = 33kHz$ $G_2 = 1500$ U_{met0}/V	$f_2 = 100kHz$ $G_2 = 1000$ U_{met0}/V
30	0.092 ± 0.004	0.27 ± 0.01	0.216 ± 0.01	0.23 ± 0.01	0.193 ± 0.01	0.265 ± 0.001
100	0.067 ± 0.002	0.24 ± 0.008	0.21 ± 0.01	0.23 ± 0.009	0.192 ± 0.01	0.265 ± 0.001
300	0.027 ± 0.01	0.18 ± 0.01	0.197 ± 0.003	0.223 ± 0.01	0.191 ± 0.01	0.265 ± 0.001
1000	0.004 ± 0.001	0.06 ± 0.01	0.151 ± 0.004	0.206 ± 0.002	0.188 ± 0.015	0.265 ± 0.001
3000	0.001 ± 0.001	0.008 ± 0.001	0.061 ± 0.001	0.163 ± 0.001	0.176 ± 0.001	0.25 ± 0.01

Table 2: Amp Noise

Compare [1, Section 1.3].

2.1.2 Johnson Noise Dependence on Bandwidth

Then we measured how the (total) noise depends on bandwidth $(f_2 - f_1)$.

f_1/Hz	$f_2 = 0.33kHz$ $G_2 = 10000$ U_{met}/V	$f_2 = 1kHz$ $G_2 = 10000$ U_{met}/V	$f_2 = 3.3kHz$ $G_2 = 5000$ U_{met}/V	$f_2 = 10kHz$ $G_2 = 3000$ U_{met}/V	$f_2 = 33kHz$ $G_2 = 1500$ U_{met}/V	$f_2 = 100kHz$ $G_2 = 1000$ U_{met}/V
30	0.29 ± 0.02	0.9 ± 0.05	0.762 ± 0.01	0.825 ± 0.01	0.7 ± 0.01	0.955 ± 0.01
100	0.22 ± 0.02	0.84 ± 0.05	0.745 ± 0.004	0.82 ± 0.01	0.695 ± 0.01	0.953 ± 0.01
300	0.09 ± 0.01	0.65 ± 0.02	0.7 ± 0.03	0.8 ± 0.01	0.69 ± 0.03	0.953 ± 0.01
1000	0.009 ± 0.006	0.23 ± 0.01	0.537 ± 0.01	0.745 ± 0.01	0.675 ± 0.01	0.948 ± 0.01
3000	0.002 ± 0.001	0.024 ± 0.003	0.222 ± 0.004	0.583 ± 0.01	0.64 ± 0.01	0.934 ± 0.01

Table 3: Noises

Compare [1, Section 1.5].

Then we calculated the Johnson Noise component (by interval arithmetic) and finally the Boltzmann constant k_B from all this:

f_1/Hz	$f_2 = 0.33kHz$ $G_2 = 10000$ $\Delta U_{met}/V$	$f_2 = 1kHz$ $G_2 = 10000$ $\Delta U_{met}/V$	$f_2 = 3.3kHz$ $G_2 = 5000$ $\Delta U_{met}/V$	$f_2 = 10kHz$ $G_2 = 3000$ $\Delta U_{met}/V$	$f_2 = 33kHz$ $G_2 = 1500$ $\Delta U_{met}/V$	$f_2 = 100kHz$ $G_2 = 1000$ $\Delta U_{met}/V$
30	$5.5 \cdot 10^{-14}$	$1.75 \cdot 10^{-13}$	$6.07 \cdot 10^{-13}$	$1.84 \cdot 10^{-12}$	$6.26 \cdot 10^{-12}$	$1.92 \cdot 10^{-11}$
100	$4.25 \cdot 10^{-14}$	$1.67 \cdot 10^{-13}$	$5.94 \cdot 10^{-13}$	$1.82 \cdot 10^{-12}$	$6.21 \cdot 10^{-12}$	$1.91 \cdot 10^{-11}$
300	$1.75 \cdot 10^{-14}$	$1.31 \cdot 10^{-13}$	$5.59 \cdot 10^{-13}$	$1.78 \cdot 10^{-12}$	$6.16 \cdot 10^{-12}$	$1.91 \cdot 10^{-11}$
1000			$4.29 \cdot 10^{-13}$	$1.66 \cdot 10^{-12}$	$6.01 \cdot 10^{-12}$	$1.9 \cdot 10^{-11}$
3000			$1.79 \cdot 10^{-13}$	$1.3 \cdot 10^{-12}$	$5.73 \cdot 10^{-12}$	$1.9 \cdot 10^{-11}$

Table 4: Johnson Noise Component (calculated)

f_1/Hz	$f_2 = 0.33kHz$ $G_2 = 10000$ $k_B/(J/K)$	$f_2 = 1kHz$ $G_2 = 10000$ $k_B/(J/K)$	$f_2 = 3.3kHz$ $G_2 = 5000$ $k_B/(J/K)$	$f_2 = 10kHz$ $G_2 = 3000$ $k_B/(J/K)$	$f_2 = 33kHz$ $G_2 = 1500$ $k_B/(J/K)$	$f_2 = 100kHz$ $G_2 = 1000$ $k_B/(J/K)$
30	$(1.42 \pm 0.267)10^{-23}$	$(1.39 \pm 0.224)10^{-23}$	$(1.42 \pm 0.147)10^{-23}$	$(1.41 \pm 0.142)10^{-23}$	$(1.46 \pm 0.184)10^{-23}$	$(1.48 \pm 0.181)10^{-23}$
100	$(1.41 \pm 0.299)10^{-23}$	$(1.43 \pm 0.232)10^{-23}$	$(1.42 \pm 0.133)10^{-23}$	$(1.41 \pm 0.14)10^{-23}$	$(1.45 \pm 0.183)10^{-23}$	$(1.48 \pm 0.181)10^{-23}$
300	$(1.45 \pm 0.556)10^{-23}$	$(1.42 \pm 0.186)10^{-23}$	$(1.43 \pm 0.189)10^{-23}$	$(1.41 \pm 0.143)10^{-23}$	$(1.46 \pm 0.241)10^{-23}$	$(1.48 \pm 0.181)10^{-23}$
1000			$(1.42 \pm 0.147)10^{-23}$	$(1.42 \pm 0.126)10^{-23}$	$(1.45 \pm 0.199)10^{-23}$	$(1.48 \pm 0.181)10^{-23}$
3000			$(1.45 \pm 0.143)10^{-23}$	$(1.41 \pm 0.131)10^{-23}$	$(1.47 \pm 0.162)10^{-23}$	$(1.51 \pm 0.205)10^{-23}$

Table 5: k_B (calculated)

$k_B = (1.42 \pm 0.1) \cdot 10^{-23} \frac{\mathrm{J}}{\mathrm{K}}$
We calculated this overall k_B by using interval intersection.

2.1.3 Johnson Noise Dependence on Resistance

Then we measured how the Johnson noise depends on resistance.

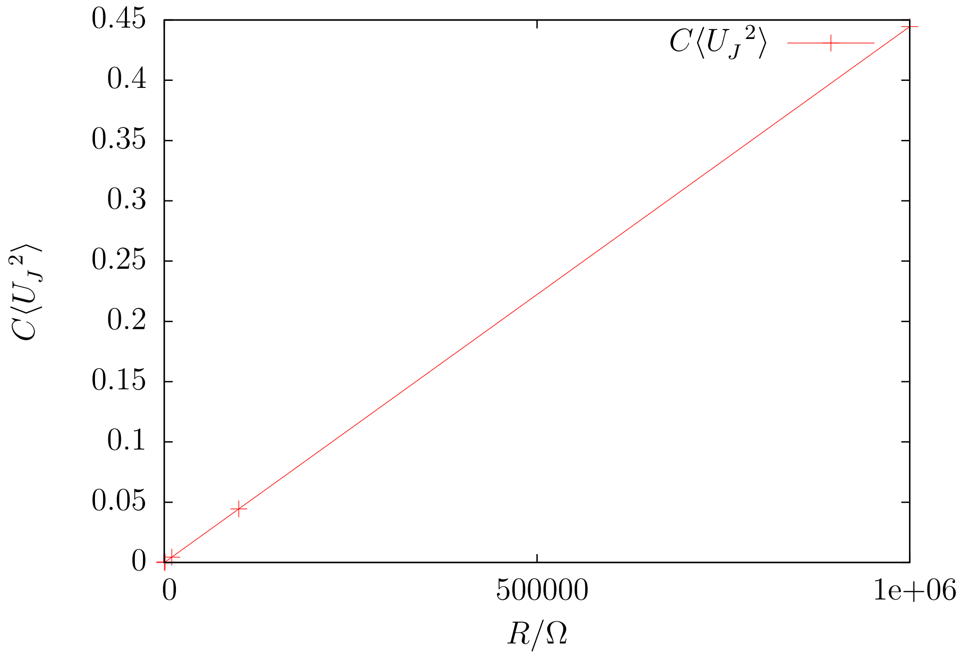
Selected bandwidth was ($f_1 = 100Hz, f_2 = 100kHz$) and main amplifier gain was $G_2 = 1500$. Therefore, effective bandwidth $\Delta f = 110961Hz$ and $U_{met0} = 0.265V$ for $G_2 = 1000$ due to amp noise (see table above).

$$\frac{10VU_{met}}{{G_1}^2(1500Hz)^2} = \langle {U_J}^2 \rangle + \langle {U_N}^2 \rangle$$
$$\frac{10VU_{met0}}{{G_1}^2(1000Hz)^2} = \langle {U_N}^2 \rangle$$
$$\frac{10V}{{G_1}^2}\left(\frac{U_{met}}{(1500Hz)^2} - \frac{U_{met0}}{(1000Hz)^2}\right) = \langle {U_J}^2 \rangle$$
$$\frac{10V}{{G_1}^2}\left(\frac{U_{met}}{(1500Hz)^2} - \frac{0.265V}{(1000Hz)^2}\right) = \langle {U_J}^2 \rangle$$

R/Ω	$U_{met}/(mV)$
100	43.4
1000	53.4
10000	153.5
100000	903
1000000	1024

Table 6: Noises Dependence on Resistance

We verified the linear dependence on R in our (small) sample (C is an arbitrary constant):



Compare [1, Section 1.4].

2.1.4 Two-temperature Johnson Noise Measurement

We selected the following settings:

$$f_1 := 100\text{Hz}$$
$$f_2 := 3.3\text{kHz}$$
$$\Delta f = 3554\text{Hz}$$

Then we measured Johnson noise once at room temperature ($T = 295K$) and once at $T = 77K$.

Compare [1, Section 2.2].

First we measured the resistor at room temperature.

$$T := 295\text{K}$$

T/K	$R = 12\Omega$ $G_2 = 400$ U_{met}/V	$R = 12\Omega$ $G_2 = 5000$ U_{met}/V	$R = 9.97k\Omega$ $G_2 = 400$ U_{met}/V	$R = 9.97k\Omega$ $G_2 = 5000$ U_{met}/V	$R = 102.1k\Omega$ $G_2 = 400$ U_{met}/V	$R = 102.1k\Omega$ $G_2 = 5000$ U_{met}/V
295	0.001	0.215	0.00465	0.745	0.8335	0.855

Table 7: Noises

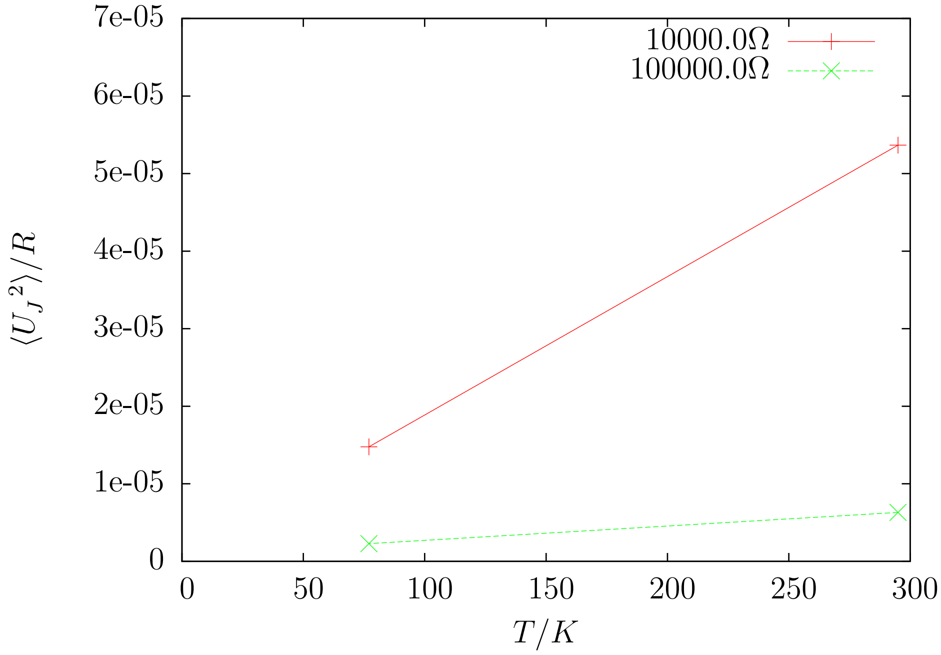
Then we measured the resistor in thermal conduct with liquid nitrogen,

$$T := 77\text{K}$$

T/K	$R = 11\Omega$ $G_2 = 400$ U_{met}/V	$R = 10.15k\Omega$ $G_2 = 5000$ U_{met}/V	$R = 99.1k\Omega$ $G_2 = 2000$ U_{met}/V
77	0.218	0.360	0.260

Table 8: Noises

We interpolate absolute zero temperature:



2.2 Conclusion and Summary

We conclude that the Boltzmann constant k_B is:
$k_B = (1.42 \pm 0.1) \cdot 10^{-23} \frac{\mathrm{J}}{\mathrm{K}}$
Also, with high enough resistances it can be seen that the origin of the Kelvin scale is well-chosen and that specifying the temperature in Kelvin is useful for blackbody radiation.

References

[1] David Van Baak, Georg Herold, Jonathan Reichert, Johannes Majer, Stefan Putz, Christian Koller: Noise Fundamentals NF1-A

[2] H. Nyquist: Thermal Agitation of Electric Charge in Conductors